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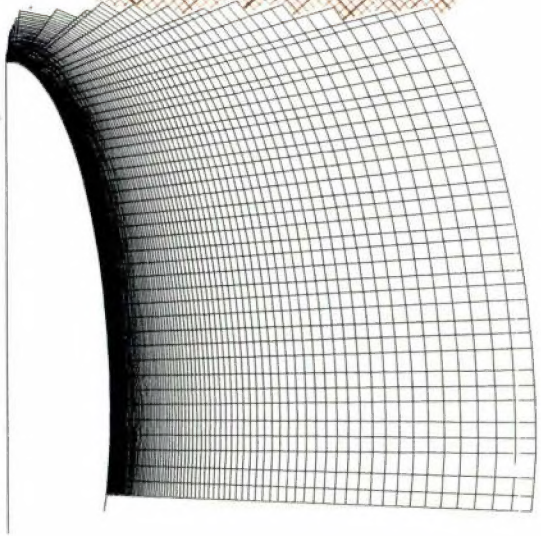

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Direcção, Administração e Publicidade
AEIST
Av. Rovisco Pais
1000 Lisboa
Portugal

Telefones directos
8481018 - 8489323
Extensão IST
1248
Fax
899242 (a/c AEIST)
E-Mail
aeistdoc@eta.ist.rccn.pt

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Conselho Directivo do Intituto Superior Técnico

EDITORIAL

Com ligeiras diferenças, a regularidade a que nos impusemos tem sido respeitada.

Para isso muito tem contribuído a abertura e generosa ajuda do Conselho Directivo do I.S.T..

Lamenta-se a falta de original. Se, por um lado, temos recebido artigos de grande valor científico, o que é facto é que, atendendo à actividade de I&D do I.S.T., o seu número é manifestamente escasso.

Saliente-se que, mais do que a nota científica, aquilo que tem rareado até à quase ausência é o artigo técnico-científico dando conta dos avanços da Técnica e Ciência nos domínios curriculares dos cursos professados no I.S.T..

No entanto, completamente diferente tem sido a reacção de todos aqueles que comparticipam no número especial da *TÉCNICA*, integrado na comemoração dos 80 anos do I.S.T. e da A.E.I.S.T..

Este facto dá-nos esperanças relativamente àquele original que é possível e não aparece.

Abreu, Rodrigo de (1990). *The First Principle of Thermodynamics and the Non-Separability of the Quantities "Work" and "Heat"*, Técnica.

Relativamente ao artigo indicado recebeu-se, com pedido de publicação, uma carta do Prof. J. Dias de Deus. A Técnica solicitou ao Prof. Rodrigo de Abreu a eventual apresentação de uma resposta que seria publicada em conjunto com a carta do Prof. Dias de Deus.

Em conformidade, reproduzem-se as facsimiles das referidas cartas.

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Exmº Senhor

Prof. Manuel José Abreu Faro

Director Científico da Revista "TÈCNICA"

IST

Lisboa, 8 de Julho de 1991

ref: 112/1.2-141

Assunto: **"The First Principle of Thermodynamics and The Non Separability of the Quantities "Work" and "Heat" ", Técnica, número único, 43, 1990.**

Exmº Senhor Professor,

Tendo vários colegas chamado a minha atenção para uma incorrecção contida no artigo acima referido, gostaria de esclarecer o seguinte:

No artigo, o autor considera um gás isolado num recipiente dividido por uma parede termicamente isoladora que se pode mover. Num lado tem-se gás à pressão P_1 e à temperatura T_1. No outro tem-se P_2 e T_2. De início $P_1=P_2=P$ e $T_1 \neq T_2$.

O autor argumenta que como $T_1 \neq T_2$ e portanto a energia cinética média de uma molécula dum lado é diferente da energia cinética média do outro, os choques vindos dum lado, sendo mais energéticos, fariam mover a parede.

Realmente, não é assim. Como as temperaturas são diferentes, $T_1 \neq T_2$, e as pressões são iguais, $P_1=P_2=P$, isso quer dizer que as densidades, dum lado e doutro, são diferentes, $\rho_1 \neq \rho_2$ (basta lembrar o modelo dos gases perfeitos!). Portanto, se é verdade que nos choques que veem dum lado há mais momento linear transportado, os choques que vêem do outro lado são mais frequentes porque há mais moléculas. No fim, tudo se compensa. A temperatura e a densidade harmonizam-se para dar exactamente a mesma pressão — como tinha de ser.

Em conclusão: a diferença de temperatura não faz mover paredes e a energia transferida sob forma de calor não se confunde com trabalho.

Julgo que este esclarecimento deveria ser publicado na TÉCNICA.

Com os melhores cumprimentos,

J. Dias de Deus

(Professor do IST)

C.Cópia: Profs: Rodrigo Abreu, A.Brotas, A. Mourão, G. Martinho, J. Delgado Domingos, A. Quintela, E.G.Santos.

Lisboa, 17 de Julho de 1991

Exmo. Senhor
Professor Manuel José de Abreu Faro
Director Científico da Revista "TÉCNICA"
IST

Enviou-me V. Exa. fotocópia duma carta que recebeu do Professor J. Dias de Deus solicitando eventual resposta para em conjunto com essa carta ser publicada na "TÉCNICA".

Agradecia que na "TÉCNICA" se publicasse integralmente o que abaixo se transcreve:

Resposta às observações feitas pelo Professor J. Dias de Deus

Foi-me enviada a seguinte objecção que se pode reduzir ao seguinte pressuposto e afirmações:

1. Se a pressão $p_1 = p_2$ então a parede não se move.

2. O pressuposto $T_1 \neq T_2$ não implica $p_1 \neq p_2$ dado que n_2 pode ser diferente de n_1 (n densidade de moléculas) e deste modo compensar a diferença de momento linear em cada colisão.

3. Logo, dado 2, a parede não se move, "como tinha de ser".

Ora a afirmação categórica de que a parede não se move, que se pretende final - "como tinha de ser", deve-se ao pressuposto 1 à partida admitido, considerado como óbvio, mas que não passa tão somente da afirmação de igualdade de pressões (a conclusão foi confundida com a premissa).

Feynman (ref. 7 do referido artigo) na análise que fez desta questão admite que $p_1 = p_2$ e $T_1 \neq T_2$ de acordo com 2:

"So we can arrange that the pressures are equal; that just means that the internal energies per unit volume are equal, or that the numbers n times the average kinetic energies on each side are equal... So far, all we know is that the numbers

$$n_1 < m_1 V_1^2 /2 = n_2 < m_2 V_2^2 /2 >$$

from (39.8), because the pressures are equal".

Como Feynman salienta, a pressão traduz um valor médio e valores médios iguais não bloqueiam a parede.

"That is, that the intermediate piston does not receive a steady pressure; it wiggles, just like the eardrum that we were first talking about, because the bangings are not absolutely uniform. There is not a perpetual, steady pressure, but a tattoo - the pressure varies, and so the thing jiggles.

So, as a result of the collisions, the piston finds itself jiggling, jiggling, jiggling, and this shakes the other gas - it gives

energy to the other atoms, and they build up faster motions, until they balance the jiggling that the piston is giving to them. The system comes to some equilibrium where the piston is moving at such a mean square speed that it picks up energy from the atoms at about the same rate as it puts energy back into them".

Limitei-me a usar o modelo da parede móvel nas condições e de acordo com a análise de Feynman (a de que a parede se move até as temperaturas serem iguais) para retirar dessa análise a consequência de que as grandezas $dw = -pdV$ e $dQ = TdS$ não têm o significado físico que por vezes se lhes atribui, nomeadamente no caso presente $dQ = TdS \neq 0$! (ver Adenda).

Conclusão:

Nos termos referidos, a parede, depois de desbloqueada, estará sempre em movimento, deslocando-se nas condições de temperaturas diferentes para uma posição média diferente da inicial mas onde exibirá sempre flutuações, uma vez que está desbloqueada.

Adenda

Embora tenha respondido cabalmente às observações do Prof. Dias de Deus, vou seguidamente, para um completo esclarecimento deste assunto, apresentar a formulação de um problema e mostrar que se obteria uma solução incorrecta se se aplicasse a condição $dQ = 0$ à parede móvel.

Um recipiente de volume V está dividido internamente em 3 compartimentos de igual volume, de início isolados térmica e mecanicamente.

A	B	C

A: 2 moles de Hélio, temperatura $70^{\circ}C$

B: 3 moles de Argon, temperatura $70^{\circ}C$

C: 2 moles de Hélio, temperatura $220^{\circ}C$

a) Quais as energias do Hélio e do Argon?

b) Torna-se a parede A/B móvel. Qual a relação final entre os volumes V_A e V_B?

A solução da alínea b, a que nos interessa, é a seguinte:

De

$$dU = dW + dQ = -pdV + dQ$$

vem

$$dU = -pdV. \quad (1)(dQ = 0)$$

Dado que

$$p = \frac{2}{3}\,\frac{U}{V} \quad \text{(gás monoatómico)}$$

de (1) vem

$$UV^{2/3} = \text{const.} \qquad (2)$$

$$pV^{5/3} = \text{const.} \qquad (3)$$

Logo

$$P_{Af}\,V_{Af}^{\;5/3} = P_A\,V_A^{\;5/3} \qquad (4)$$

$$P_{Bf}\,V_{Bf}^{\;5/3} = P_B\,V_B^{\;5/3} \qquad (5)$$

Impondo

$$P_{Af} = P_{Bf}$$

vem

$$\frac{V_{Af}}{V_{Bf}} = \left(\frac{P_A\,V_A^{\gamma}}{P_B\,V_B^{\gamma}}\right)^{1/\gamma} \qquad (6)$$

em que $\gamma = \frac{5}{3}$.

(6) conduz ao seguinte absurdo:

Dado que

$$V = V_A + V_B = V_{Af} + V_{Bf},$$

determina-se V_{Af} e V_{Bf} .

Ora dado que

$$UV^{2/3} = \text{const.},$$

conclui-se que os valores U_{Af} e U_{Bf} não satisfazem à equação de conservação de energia

$$U = U_A + U_B \neq U_{Af} + U_{Bf},$$

como se poderá facilmente verificar.

Com os melhores cumprimentos,

Rodrigo de Abreu

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As Equações de Maxwell
Da sua escrita original às formas diferenciais*

Manuel José de Abreu Faro

Centro de Electrodinâmica do INIC. Departamento de Electrotecnia e Computadores, IST, UTL.

RESUMO

As equações de Maxwell descrevem exaustivamente o Campo Electromagnético para os Corpos em Repouso.

Foi Hertz quem, sistematicamente, abordou pela primeira vez o processo dos corpos em movimento. Não teve êxito mas abriu um processo que, continuado de modo principal por Lorentz, apenas se esclareceu com Einstein no contexto da Teoria da Relatividade Restrita, 1905.

Em 1908, Minkowski apresenta uma formulação que, em certa medida, resolve completamente o problema da Electrodinâmica dos Meios Materiais em Movimento. Aí se postula a covariância das Equações de Maxwell.

Nessa formulação, Minkowski restringia o carácter tensorial dessas equações às Transformações de Lorentz.

Postulando a completa covariância das Equações de Maxwell, consegue-se tratar, com suficiente generalidade e rigor, o problema dos corpos acelerados.

A formulação de Minkowski é extraordinariamente elegante e a Análise Tensorial faculta-nos meios poderosos para o estudo da Electrodinâmica dos Corpos em Movimento.

Nos últimos anos, e para a mesma finalidade, tem-se difundido o recurso às Formas Diferenciais.

Contribuir para essa difusão é o objecto fundamental do presente trabalho.

ABSTRACT

Maxwell's Equations exhaustively describe the Electromagnetic Field for the Bodies at Rest.

The problem of moving bodies was systematically approached for the first time by Hertz. He was not successful but set a trend, and the problem, mainly pursued by Lorentz, would finally be solved by Einstein within the Special Theory of Relativity.

In 1908, Minkowski presents a formulation which can be said to completely solve the problem of Electrodynamics of Moving Bodies.

In Minkowski's formulation, Maxwell's equations were only ascribed a tensorial character for Lorentz Transformations.

By postulating the complete covariance of Maxwell's Equations, the problem of the accelerated bodies can be treated with rigour and in fairly general validity conditions.

Minkowski's formulation is extremely elegant and Tensorial Analysis provides a powerful means for the study of Electrodynamics of Moving Bodies.

In recent years, Differential Forms have been more and more used with the same purpose.

To contribute to an even wider knowledge and use of these forms is the object of this work.

1 As Equações de Maxwell. A sua escrita original

Ocupa-se o presente trabalho das Equações de Maxwell.

Em certa medida, da sua perenidade formal e decorrente e natural covariância.

No domínio da Electricidade e do Magnetismo a primeira formulação quantitativa deve-se a Coulomb que descobriu a lei da atracção e repulsão das cargas eléctricas em 1785.

Essa descoberta insere-se numa sequência frutuosa para a Electricidade e Magnetismo que decorreu de 1785 a 1789.

Estamos assim a dois séculos das primeiras leis quantitativas da Electricidade e do Magnetismo.

As Equações de Maxwell não se impuseram de imediato.

As teorias de acção a distância imperavam ainda e, no domínio do Electromagnetismo, Weber e Neumann tinham desenvolvido teorias que incluíam, em si, o fenómeno electrostático e o fenómeno electromagnético.

Foi Hertz que em 1888 evidenciou a existência das ondas electromagnéticas, previstas teoricamente por Maxwell. É a partir daí que a teoria de Maxwell é definitivamente aceite.

Estes trabalhos correspondem a uma série que decorreu de 1885 a 1889, 100 anos depois dos trabalhos de Coulomb.

As equações de Maxwell descrevem exaustivamente o Campo Electromagnético dos Corpos em Repouso.

Uma coisa são as equações de Maxwell e a sua prodigiosa perfeição. Outra coisa é a atitude de Maxwell e o modo como interpretava a sua teoria. Para Maxwell,

* Comunicação à Classe de Ciências da Academia das Ciências de Lisboa em 15 de Junho de 1990.

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a interpretação correcta das suas equações deveria suportar-se num mecanismo subordinado às leis de Newton.

Maxwell não conhecia a natureza desse mecanismo mas apercebeu-se claramente que para satisfazer as equações teria que associar ao campo energia, que assim existiria onde houvesse campo electromagnético.

Não entraremos nesses pormenores, apenas desejamos salientar que Maxwell, ainda que recorrendo a modelos e mecanismos discutíveis, conseguiu escrever equações que estavam profeticamente certas. Seriam as leis da Mecânica de Newton que necessitariam de correcção no advento da Teoria da Relatividade Restrita.

As equações de Maxwell; essas, estavam certas.

No entanto, de Maxwell a Einstein, quantas ideias, quantos esforços foi necessário despender.

Quando Boltzmann tomou conhecimento das equações de Maxwell, saudou-as com um verso de Goethe pertencente ao primeiro solilóquio de Fausto.

Was es ein Gott, der diese Zeichen schrieb?

Em português: Seria um deus quem desenhou tais signos? (Verso 454, tradução do Fausto, de Goethe, por Agostinho d'Ornellas, Edição ao cuidado de Paulo Quintela, Atlântida, Coimbra, 1958.).

A primeira comunicação de Maxwell, sobre o Electromagnetismo, data de 1855: "On Faraday's Lines of Force".

Em 1861-62 surge o seu segundo trabalho: "On Physical Lines of Force".

Finalmente, em 8 de Dezembro de 1864, apresenta à Royal Society a comunicação "A Dynamical Theory of the Electromagnetic Field".

As equações, que ficariam universalmente conhecidas por Equações de Maxwell, constam, já, dessa comunicação.

Foram apresentadas na ordem que a seguir indicamos:

$$C = J + \frac{\partial D}{\partial t} \qquad (A)$$

$$\mu H = rot\, A \qquad (B)$$

$$rot\, H = C \qquad (C)$$

$$E = v \times \mu H - \frac{\partial A}{\partial t} - grad\, \psi \qquad (D)$$

$$D = \varepsilon E \qquad (E)$$

$$J = \sigma E \qquad (F)$$

$$div\, D = \rho \qquad (G)$$

$$div\, J + \frac{\partial \rho}{\partial t} = 0 \qquad (H)$$

A forma era outra. Cada uma das equações vectoriais aparecia desdobrada em três componentes segundo x, y e z.

Também os símbolos eram outros e bem assim o sistema de unidades adoptado.

Heaviside (1850-1925), que contribuiu de modo significativo na Matemática, na Física e na Engenharia Electrotécnica, simplificou as equações de Maxwell em dois aspectos distintos:

No que respeita à Física deu-lhes uma forma simétrica, fazendo intervir cargas eléctricas e cargas magnéticas, correntes de condução eléctricas e correntes de condução magnéticas e ainda correntes de convecção eléctricas e magnéticas.

À corrente de deslocamento $\partial D/\partial t$ contrapunha a corrente magnética de deslocamento $\partial B/\partial t$.

A dualidade das equações de Maxwell ressalta assim de modo claro e completo.

Embora, e até hoje, o magnetismo verdadeiro não se tenha revelado, a dualidade das equações de Maxwell tem larguíssimas aplicações, nomeadamente, no domínio da Propagação e Radiação das Ondas Electromagnéticas.

No que respeita ao aspecto matemático, introduziu a análise vectorial de onde, além do mais, resultou a escrita das equações de Maxwell em função dos operadores diferenciais do cálculo vectorial. Contribuição semelhante se ficou também devendo a Gibbs (1839-1903).

Ainda na Matemática, Heaviside foi o criador do cálculo operacional, que se constituiu da maior importância para a Electrotecnia e, de um modo geral, para a Física e Engenharia. Introduziu o uso da variável complexa na análise associada a campos harmónicos.

Foi com o título de "Sobre a Electrodinâmica dos Corpos em Movimento" que Einstein, em 1905, apresentou a Teoria da Relatividade Restrita.

No entanto, não entrou no pormenor das equações referentes a meios materiais.

Minkowski, em 1908, em plena posse do "princípio de relatividade" foi quem finalmente resolveu o problema da Electrodinâmica dos Meios em Movimento.

Assim, ou quase assim, nos diz sobre o assunto Sommerfeld.

Fundamentalmente, trata-se de uma teoria macroscópica estruturada a partir de quatro entidades E, B, D e H, duas a duas constituindo tensores anti-simétricos de segunda ordem.

No entanto, o ponto de partida são as equações de Maxwell, na forma que lhe foi conferida por Heaviside e Hertz, inteiramente equivalente ao que Maxwell escrevera em 1864 mas, na realidade, apenas resolvendo o problema dos corpos em repouso.

2 Formulação de Minkowski e sua Generalização à Electrodinâmica dos Sistemas Acelerados

Na formulação de Minkowski postula-se a covariância das Equações de Maxwell, quando escritas na forma indefinida,

$$\nabla \times E = -\frac{\partial B}{\partial t} \qquad \nabla . B = 0 \qquad (1a, b)$$

$$\nabla \times H = J + \frac{\partial D}{\partial t} \qquad \nabla . D = \rho \qquad (1c, b)$$

e sob uma Transformação de Lorentz.

Introduzindo o Tensor Campo Electromagnético

$$F_{ik}=\begin{bmatrix} 0 & E_x & E_y & E_z \\ -E_x & 0 & -cB_z & cB_y \\ -E_y & cB_z & 0 & -cB_x \\ -E_z & -cB_y & cB_x & 0 \end{bmatrix} \quad (2a)$$

o Tensor Excitação

$$G^{ik}=\begin{bmatrix} 0 & -cD_x & -cD_y & -cD_z \\ cD_x & 0 & -H_z & H_y \\ cD_y & H_z & 0 & -H_x \\ cD_z & -H_y & H_x & 0 \end{bmatrix} \quad (2b)$$

e ainda o Quadrivector Corrente Eléctrica

$$J^i=(c\rho, J_x, J_y, J_z) \quad (2c)$$

as Equações de Maxwell assumem a forma

$$\partial_i F_{jk}+\partial_k F_{ij}+\partial_j F_{ki}=0 \quad (3a)$$

$$\partial_k G^{ki}=J^i \quad (3b)$$

em que

$$\partial_i=\frac{\partial}{\partial x^i} \qquad x^0=ct \qquad x^1=x \qquad x^2=y \qquad x^3=z \quad (3c)$$

Definindo o dual $^*F^{ik}$ de F^{ik} como

$$^*F^{ik}=\frac{1}{2\sqrt{-g}}\varepsilon^{iklm}F_{lm} \quad (4)$$

em que ε^{iklm} é o indicador de Levi Civita e g o determinante $|g_{ik}|$, as Equações de Maxwell podem escrever-se

$$\partial_k\, ^*F^{ki}=0 \quad (5a, b)$$

$$\partial_k G^{ki}=J^i \quad (5c, b)$$

A partir de (4) e tendo em atenção que o indicador de Levi Civita vale $(+1)$ para qualquer permutação par de índices todos diferentes, (-1) para uma permutação ímpar e se anula sempre que quaisquer dois índices sejam iguais, é fácil obter o dual $^*F^{ik}$

$$^*F^{ik}=\begin{bmatrix} 0 & -cB_x & -cB_y & -cB_z \\ cB_x & 0 & E_z & -E_y \\ cB_y & -E_z & 0 & E_x \\ cB_z & E_y & -E_x & 0 \end{bmatrix} \quad (6)$$

Note-se que o indicador de Levi Civita não é um tensor.

Tensor é a grandeza de componentes contravariantes e covariantes, respectivamente, dadas por

$$e^{iklm}=+\frac{1}{\sqrt{-g}}\varepsilon^{iklm} \qquad e_{iklm}=-\sqrt{-g}\;\varepsilon_{iklm} \quad (7a, b)$$

Há autores que definem (7a) e (7b) com sinal contrário ao que se indica.

F_{ik} e G^{ik} podem ser interpretados como elementos de matrizes mas isso não significa que as equações de Maxwell tenham carácter tensorial.

Postulando que assim é, estamos imediatamente a vincular os campos F_{ik} e G^{ik} a determinadas leis de transformação.

Minkowski apenas considerava a Transformação de Lorentz que, reportando-se a coordenadas cartesianas, nos permite interpretar ∂_i como as componentes covariantes de um quadrivector.

Utilizando coordenadas curvilíneas seria fácil verificar que, num dado referencial, as Equações de Maxwell se podem escrever

$$D_k\, ^*F^{ki}=0 \quad (8a)$$

$$D_k G^{ki}=J^i \quad (8b)$$

em que D_k significa derivação covariante em relação a x^k.

Para transformações espaciais, coordenadas cartesianas (x, y, z), coordenadas esféricas (r, θ, ϕ) etc., facilmente se verifica que é lícito escrever (8a, b).

No entanto, tratando-se de transformações quaisquer no espaço-tempo, já a questão não é evidente e, assim, traduz um postulado sujeito a confirmação experimental.

Nessa atitude, as equações de Maxwell (8a, b) são covariantes em qualquer transformação e passamos a dispor de um processo de abordar formalmente o estudo de sistemas acelerados.

Para domínios restritos do espaço-tempo e desde que a energia electromagnética não altere a curvatura do espaço, isso é válido.

Isto é importante e tem sido utilizado na Electrodinâmica dos Sistemas Acelerados.

Evidentemente que, conferindo carácter tensorial a F^{ki}, G^{ki} e J^i, as leis de transformação dos campos e correntes são as que decorrem da lei tensorial e o que daí resulta está, como se disse, sujeito a confirmação experimental.

3 As Formas Diferenciais

3.1 Introdução

As Formas Diferenciais funcionam no âmbito do Cálculo Diferencial Exterior que se suporta fundamentalmente em dois conceitos básicos: um de carácter algébrico e que é o de Produto Exterior, outro no âmbito da Análise Diferencial e que é o de Diferencial Exterior.

As Formas Diferenciais foram introduzidas no princípio deste século. A descoberta das suas potencialidades ficaram-se devendo, de modo principal, às contribuições de E. Cartan.

Trata-se pois de um conhecimento bem dominado pela Análise Matemática.

Quanto importa à Física, e também à Engenharia, é aproveitar da elegância formal e simplicidade de trato

que as Fórmulas Diferenciais facultam à resolução de problemas dessas áreas.

As Equações de Maxwell assumem uma escrita simples quando se recorre às Formas Diferenciais.

Isso deve-se, em última análise, ao carácter tensorial das Equações de Maxwell e ao facto de subjacente à Teoria das Formas Diferenciais estar o Cálculo Tensorial.

Explica-se, assim, o interesse crescente da Electrodinâmica pelas Formas Diferenciais, nomeadamente, a Electrodinâmica dos Corpos em Movimento.

Limitamo-nos a expor alguns aspectos essenciais, suficientes, julgamos, para apreender o que é característico da Teoria das Formas Diferenciais.

A linguagem é a de quem aplica e, assim, não assume a forma típica dos actuais tratados de Cálculo Diferencial Exterior.

3.2 Produto Exterior

Consideremos o vector

$$A = A_i e^i \qquad i = 1, 2, \ldots, n \tag{9a}$$

expresso na base $[e^i]$ e de acordo com a convenção de Einstein.

Consideremos também o vector

$$B = A_k e^k \qquad k = 1, 2, \ldots, n \tag{9b}$$

O produto exterior goza das propriedades distributiva e associativa mas é anti-comutativo. Exprime-se pelo símbolo $\wedge$.

Pela propriedade anti-comutativa teremos, sucessivamente,

$$e^i \wedge e^k = -e^k \wedge e^i \tag{9c}$$

e

$$e^i \wedge e^i = 0 \tag{9d}$$

Pela propriedade distributiva

$$A \wedge B = A_i A_k e^i \wedge e^k \tag{9e}$$

Pela propriedade associativa

$$e^i \wedge e^k \wedge e^j = (e^i \wedge e^k) \wedge e^j = e^i \wedge (e^k \wedge e^j) \tag{9f}$$

Conjugando (9d) com a propriedade associativa imediatamente se conclui que um produto em que existam dois factores iguais é nulo.

3.3 Forma exterior de grau-p

Será, por definição,

$$\alpha = \frac{1}{p!} \alpha_{i_1 i_2 \ldots i_p} e^{i_1} \wedge e^{i_2} \ldots \wedge e^{i_p} \tag{10a}$$

em que $\alpha_{i_1 i_2 \ldots i_p}$ são as componentes covariantes de um tensor completamente anti-simétrico.

Em (10a) recorre-se uma vez mais à convenção de Einstein e num espaço a n dimensões

$$i_1, i_2 \ldots, i_p = 1, 2, \ldots n \tag{10c}$$

Pelas propriedades do produto exterior

$$e^{i_1} \wedge e^{i_2} \ldots \wedge e^{i_p} = (-1)^I e^{j_1} \wedge e^{j_2} \ldots \wedge e^{i_p} \tag{10d}$$

em que $j_1 j_2 \ldots j_p$ representa uma permutação de $i_1 i_2 \ldots i_p$ satisfazendo à condição $j_1 < j_2 < \ldots < j_p$ e I representa o número total de inversões que se observam em $i_1 i_2 \ldots i_p$.

Como o tensor é anti-simétrico, também se verifica

$$\alpha_{i_1 i_2 \ldots i_p} = (-1)^I \alpha_{j_1 j_2 \ldots j_p} \tag{10e}$$

Em última análise, resulta que (10a) é equivalente a escrever

$$\alpha = \alpha_{|i_1 i_2 \ldots i_p|} e^{i_1} \wedge e^{i_2} \ldots \wedge e^{i_p} \tag{10f}$$

em que $|i_1 i_2 \ldots i_p|$ significa $i_1 < i_2 < \ldots < i_p$.

As componentes $\alpha_{|i_1 i_2 \ldots i_p|}$ designam-se por componentes estritas da forma exterior de grau-p.

O número dessas componentes é dado por $\binom{n}{p}$.

Num espaço a n dimensões $p \leq n$ uma vez que $i_1 i_2 \ldots i_p$ tomam valores distintos. Se dois índices fossem iguais, e isso aconteceria necessariamente se $p > n$, o produto exterior $e^{i_1} \wedge e^{i_2} \ldots \wedge e^{i_p}$ teria dois factores iguais e anulava-se.

Pode provar-se que

$$\underset{p}{\alpha} \wedge \underset{q}{\beta} = (-1)^{pq} \underset{q}{\beta} \wedge \underset{p}{\alpha} \tag{11}$$

em que α é uma forma de grau p e β uma forma de grau q.

3.4 Diferencial exterior. Formas diferenciais

A diferencial exterior de uma função escalar será, por definição,

$$df = \frac{\partial f}{\partial x^i} dx^i \tag{12a}$$

em que d se deve entender como diferencial exterior de f e diferencial exterior da variável x^i.

Se tivermos em consideração que as componentes

$$\frac{\partial f}{\partial x^1} \tag{12b}$$

se transformam como as componentes covariantes de um vector e se identificarmos a diferencial exterior de x^i com o elemento e^i da base $[e^i]$ e se, ainda e finalmente, interpretarmos $[e^i]$ como a base recíproca de $[e_i]$, o que equivale a escrever,

$$A = A_i e^i = A^i e_i \tag{12c}$$

podemos dizer que:

A função escalar é uma forma de grau zero.

A diferencial exterior de f, df, é uma forma de grau 1 expressa em dx^i.

Quanto se disse para as formas exteriores expressas na base $[e^i]$ se diz para as formas exteriores expressas na base $[dx^i]$ que, neste particular, se passam a designar por formas diferenciais.

Em (12a) df é uma forma diferencial de grau-1.

Tratando-se de uma forma diferencial de grau-p teremos

$$\alpha = \alpha_{|i_1 i_2 \ldots i_p|} dx^{i_1} \wedge dx^{i_2} \ldots \wedge dx^{i_p} \tag{13}$$

Podemos generalizar o conceito de diferencial exterior que, por definição, será

$$d\alpha = d\alpha_{|i_1 i_2 \ldots i_p|} \wedge dx^{i_1} \wedge dx^{i_2} \ldots \wedge x^{i_p} \qquad (14a)$$

Recorrendo à definição (12a) imediatamente se tem

$$d\alpha = \frac{\partial \alpha_{|i_1 i_2 \ldots i_p|}}{\partial x^j} dx^j \wedge dx^{i_1} \wedge dx^{i_2} \ldots \wedge x^{i_p} \qquad (14b)$$

Assim se conclui que a diferencial exterior de uma forma de grau p é uma forma $(p+1)$.

Poder-se-ia demonstrar que

$$d(\alpha + \beta) = d\alpha + d\beta \qquad (15a)$$

em que α e β são formas do mesmo grau. Se α for uma forma de grau p e β uma forma de grau q também se demonstra que

$$d(\underset{p}{\alpha} \wedge \underset{q}{\beta}) = d\alpha \wedge \beta + (-1)^p \alpha \wedge d\beta \qquad (15b)$$

Importante é também o Lema de Poincaré que, para qualquer $d\alpha$, se escreve

$$d(d\alpha) = d^2\alpha = 0 \qquad (15c)$$

Também se pode demonstrar o seu recíproco, isto é, se

$$d\alpha = 0 \qquad (15d)$$

então α é a diferencial exterior de uma forma β

$$\alpha = d\beta \qquad (15e)$$

4 Mudança de Variáveis

Consideremos as transformações de coordenadas

$$x = x(u, v, \omega) \qquad y = y(u, v, \omega) \qquad z = z(u, v, \omega) \qquad (16)$$

Como é conhecido do cálculo tensorial, a base $[dx, dy, dz]$ exprime-se na base $[du, dv, d\omega]$ pelas relações

$$\begin{aligned} dx &= \frac{\partial x}{\partial u} du + \frac{\partial x}{\partial v} dv + \frac{\partial x}{\partial \omega} d\omega \\ dy &= \frac{\partial y}{\partial u} du + \frac{\partial y}{\partial v} dv + \frac{\partial y}{\partial \omega} d\omega \\ dz &= \frac{\partial z}{\partial u} du + \frac{\partial z}{\partial v} dv + \frac{\partial z}{\partial \omega} d\omega \end{aligned} \qquad (17a)$$

Atendendo às propriedades do produto exterior, facilmente se conclui que:

$$dx \wedge dy = \frac{\partial(x, y)}{\partial(u, v)} du \wedge dv + \frac{\partial(x, y)}{\partial(v, \omega)} dv \wedge d\omega + \frac{\partial(x, y)}{\partial(\omega, u)} d\omega \wedge du$$

$$dx \wedge dy \wedge dz = \frac{\partial(x, y, z)}{\partial(u, v, \omega)} du \wedge dv \wedge d\omega \qquad (17c)$$

A regra generaliza-se a um espaço de dimensão qualquer e facilmente se exprime uma forma diferencial de base $[dx^i]$ na correspondente forma diferencial de base $[d\hat{x}^i]$, basta calcular os diversos jacobianos e reagrupar os termos correspondentes aos mesmos produtos exteriores.

5 Integração. Teorema de Stokes

Um modo bastante acessível de introduzir o conceito de integração de uma forma diferencial e correspondente processo de operar é dado por Misner, Thorne e Wheeler em Gravitation.

Recorre ao conceito de contracção de forma exteriores com tensores.

Ultrapassaremos a sua exposição e limitamo-nos a escrever

$$\int \alpha = \int \alpha_{|i_1 i_2 \ldots i_p|} dx^{i_1} \wedge dx^{i_2} \ldots \wedge dx^{i_p} \qquad (18a)$$

que significa um integral de uma forma sobre um determinado domínio.

Se for sobre uma curva, escrevemos as equações paramétricas

$$x^i = x^i(\lambda) \qquad (18b)$$

Se for sobre uma superfície escrevemos as equações

$$x^i = x^i(\lambda^1, \lambda^2) \qquad (18c)$$

Se for a integração sobre um domínio p-dimensional escreveremos as equações paramétricas desse domínio

$$x^i = x^i(\lambda^1, \lambda^2 \ldots, \lambda^p) \qquad (18d)$$

Então, com generalidade, o integral (18a) será dado por

$$\int \alpha = \int \alpha^*_{|i_1 i_2 \ldots i_p|} \frac{\partial(x^{i_1}, x^{i_2} \ldots x^{i_p})}{\partial(\lambda^1, \lambda^2 \ldots \lambda^p)} d\lambda^1 d\lambda^2 \ldots d\lambda^p \qquad (18e)$$

em que $\alpha^*_{|i_1 i_2 \ldots i_p|}$ significa $\alpha_{|i_1 i_2 \ldots i_p|}$ em função de $\lambda^1 \lambda^2, \ldots, \lambda^p$ e $d\lambda^1$, $d\lambda^2 \ldots$, $d\lambda^p$ devem ser interpretados como diferenciais ordinárias das variáveis $\lambda^1, \lambda^2, \ldots, \lambda^p$.

O Teorema Generalizado de Stokes tem a seguinte formulação

$$\int_V d\alpha = \int_{\partial V} \alpha \qquad (19)$$

em que α é uma forma de grau p e, consequentemente, $d\alpha$ é uma forma de grau $(p+1)$. V é um domínio $(p+1)$ dimensional limitado pelo domínio fechado ∂V que é p dimensional.

Por adequada escolha de α, o teorema generalizado de Stokes conduz aos teoremas de Green, de Ostrogradsky-Gauss e de Stokes (forma clássica).

6 As Formas Diferenciais das Equações de Maxwell

Consideremos a forma de grau 1

$$V = V_i dx^i \qquad i = 1, 2, 3 \qquad (20a)$$

A diferencial exterior será

$$dV = \frac{\partial V_i}{\partial x^k} dx^k \wedge dx^i \qquad (20b)$$

o que conduz a

$$dV=\left(\frac{\partial V_3}{\partial x^2}-\frac{\partial V_2}{\partial x^3}\right)dx^2\wedge dx^3+\left(\frac{\partial V_1}{\partial x^3}-\frac{\partial V_3}{\partial x^1}\right) \quad (20c)$$

$$dx^3\wedge dx^1+\left(\frac{\partial V_2}{\partial x^1}-\frac{\partial V_1}{\partial x^2}\right)dx^1\wedge dx^2$$

Esta forma tem por componentes estritas as componentes de *rot V*.

Consideremos a forma de grau 2

$$U=U_{23}dx^2\wedge dx^3+U_{31}dx^3\wedge dx^1+U_{12}dx^1\wedge dx^2 \quad (20d)$$

A diferencial exterior será

$$dU=\left(\frac{\partial U_{23}}{\partial x^1}+\frac{\partial U_{21}}{\partial x^2}+\frac{\partial U_{12}}{\partial x^3}\right)dx^1\wedge dx^2\wedge dx^3 \quad (20e)$$

recorrendo à definição de diferencial exterior e às propriedades do produto exterior facilmente se obtém (20e).

O termo entre parêntesis é a divergência de um vector de componentes (U_{23}, U_{31}, U_{12}).

Se identificarmos U com dV de (20c)

$$U=dV \quad (20f)$$

imediatamente se verifica que

$$dU=d(dV)=0 \quad (20g)$$

o que era de esperar pelo Lema de Poincaré.

Destes resultados se conclui que o rotacional de um vector se pode escrever sob a forma dV, em que V é uma forma de grau 1, e que a divergência de um vector se pode escrever sob a forma dU, em que U é uma forma de grau 2.

Consideremos agora as equações de Maxwell

$$rot\,E=-\frac{\partial B}{\partial t}$$

$$div\,B=0 \quad (21a)$$

$$rot\,H=J+\frac{\partial D}{\partial t}$$

$$div\,D=\rho$$

A segunda equação, pelo Lema de Poincaré e resultado obtido em (20c), conduz sucessivamente a

$$B=dA \quad (21b)$$

e

$$A=A_x\,dx+A_y\,dy+A_z\,dz \quad (21c)$$

B é uma forma de grau 2 e então E é de grau 1

$$E=E_x\,dx+E_y\,dy+E_z\,dz \quad (21d)$$

e a primeira equação de Maxwell escreve-se

$$dE=-\partial_t B \quad (21e)$$

o que, atendendo a (21b), se pode escrever

$$d(E+\partial_t A)=0 \quad (21f)$$

Pelo Lema de Poincaré teremos então

$$E+\partial_t A=-d\phi \quad (21g)$$

em que $d\phi$ é uma forma de grau 1 e ϕ uma forma de grau zero.

A quarta equação mostra-nos que D deverá ser uma forma de grau 2 e ρ uma forma de grau 3

$$D=D_x\,dy\wedge dz+D_y\,dz\wedge dx+D_z\,dx\wedge dy \quad (21h)$$

$$\rho=\rho\,dx\wedge dy\wedge dz$$

A terceira equação revela-nos que J é uma forma de grau 2 e H uma forma de grau 1.

Assim se conclui que as Equações de Maxwell se podem exprimir através das formas:

$$E=E_x\,dx+E_y\,dy+E_z\,dz$$

$$B=B_x\,dy\wedge dz+B_y\,dz\wedge dx+B_z\,dx\wedge dy$$

$$H=H_x\,dx+H_y\,dy+H_z\,dz \quad (22a)$$

$$D=D_x\,dy\wedge dz+D_y\,dz\wedge dx+D_z\,dx\wedge dy$$

$$J=J_x\,dy\wedge dz+J_y\,dz\wedge dx+J_z\,dx\wedge dy$$

$$\rho=\rho\,dx\wedge dy\wedge dz$$

assumindo a seguinte escrita:

$$dE=-\partial_t B \qquad dB=0$$

$$dH=J+\partial_t D \qquad dD=\rho \quad (22b)$$

Mais se conclui que B e E derivam dos potenciais A e ϕ

$$A=A_x\,dx+A_y\,dy+A_z\,dz \quad (22c)$$

$$\phi=\phi$$

tendo-se

$$E=-d\phi-\frac{\partial A}{\partial t} \quad (22d)$$

$$B=dA$$

As equações (22b) são independentes do sistema de coordenadas, embora na sua derivação se tenha recorrido a coordenadas cartesianas ortogonais em que se fez $x^1=x, x^2=y, x^3=z$.

Também as equações (22b) são independentes da métrica.

No entanto, se quisermos escrever as relações decorrentes das Equações de Maxwell em componentes físicas teremos que recorrer a uma métrica.

7 As Equações de Maxwell no Espaço-Tempo

Tomaremos como base de exposição as coordenadas

$$x^o=ct \qquad x^1=x \qquad x^2=y \qquad x^3=z \quad (23a)$$

Consideremos as equações de Maxwell escritas na secção anterior

$$dE=-\partial_t B \qquad dB=0$$

$$dH=J+\partial_t D \qquad dD=\rho \quad (23b)$$

multiplicando na primeira e terceira equação, exteriormente e à direita, por cdt, obtemos:

$$d^3 H \wedge dct = J \wedge dct + \partial_t cD \wedge dt \qquad (23c)$$

$$d^3 E \wedge dct = -\partial_t cB \wedge dt$$

d^3 significa a diferencial exterior calculada no espaço (x, y, z) e d^4 significa a diferencial exterior calculada no espaço-tempo (ct, x, y, z).

É fácil verificar que:

$$d^3 E \wedge dct = d^4 (E \wedge dct)$$

$$d^3 H \wedge dct = d^4 (H \wedge dct) \qquad (23d)$$

$$d^4 cB = d^3 cB + \frac{\partial cB}{\partial t} \wedge dt$$

$$d^4 cD = d^3 cD + \frac{\partial cD}{\partial t} \wedge dt$$

Notando que $d^3B = 0$ e $d^3D = \rho$, facilmente se verifica que as quatro equações de Maxwell equivalem a escrever

$$d^4 (E \wedge dct + cB) = 0 \qquad (23e)$$

$$d^4 (cD - H \wedge dct) = c\rho - J \wedge cdt$$

Definindo a forma de segundo grau F, dita Faraday,

$$F = E \wedge dct + cB \qquad (24a)$$

e uma forma de segundo grau, dita Maxwell,

$$M = cD - H \wedge dct \qquad (24b)$$

e, ainda, uma forma de grau 3, dita Corrente,

$$I = c\rho - J \wedge dct \qquad (24c)$$

as Equações de Maxwell revestem a forma, extremamente condensada,

$$dF = 0$$

$$dM = I \qquad (24d)$$

Estas equações são válidas em qualquer sistema de coordenadas da variedade espaço-tempo.

Não recorrem ao conceito de métrica, mas a métrica é necessária se quisermos conhecer as componentes físicas.

Note-se que E, B, D e H e bem assim ρ e J são dados pelas relações (22a).

A partir daqui toda a Electrodinâmica se pode construir recorrendo às Formas Diferenciais.

Dominando os teoremas fundamentais, as Formas Diferenciais facultam um tratamento elegante e seguro da Electrodinâmica Relativista.

Na bibliografia consultada indicam-se alguns livros e trabalhos básicos para o desenvolvimento de matéria que apenas afloramos no essencial.

Recorrendo ao conceito de dualidade, aumenta-se a potencialidade da teoria e o formalismo, por vezes, simplifica-se. Limitamo-nos a chamar a atenção para o facto. A definição de dual é idêntica à do Cálculo Tensorial, convindo salientar que nas Formas Diferenciais trabalhamos com componentes covariantes.

Quanto se disse, julgamos, é suficiente para habilitar à compreensão de trabalhos onde se recorre às Formas Diferenciais para o tratamento de problemas da Física, nomeadamente aqueles que respeitam à Electrodinâmica Relativista.

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Some Aspects of Cylindrically Symmetric Gravitational Cosmic Strings

Sudipto Bhattacharya and T.M. Karade

Department of Mathematics, Nagpur University Campus, Nagpur-440 010, India.

RESUMO

Com base no estudo do movimento de uma partícula-teste segundo uma geodésica, e tendo em conta os vectores de Killing admitidos pela métrica, com simetria cilíndrica, de uma corda cósmica, conclui-se que a densidade linear da corda cósmica não pode ser igual a 1/4.

ABSTRACT

From the study of the geodesic motion of a test particle and the Killing vectors admitted by the cylindrically symmetric metric of a cosmic string, one concludes that the linear density of the cosmic string can not be equal to 1/4.

1 Introduction

Gauge theories with spontaneously broken symmetries point to the fact that the universe might have undergone a series of phase transitions since the big-bang (Linde [5]). These phase transitions can give rise to a number of topological structures such as vacuum domain walls, cosmic strings and monopoles. Cosmic strings, which could arise as a random network of line-like defects and may possibly have survived to the present day, seem to be of interest (Kibble [3], [4], Zeldovich [10], Vilenkin [9], Gott [1]).

Vilenkin [6, 7, 8] has initiated the study of gravitational effects of gauge cosmic strings by calculating the gravitational field of a static, straight, cylindrically symmetric string lying along the z-axis. Using linear approximation to general relativity, he found that the presence of a string along the symmetry axis in space-time produces a conical, wedge-like singularity in the exterior, with a deficit angle proportional to the linear density of the string.

Hiscock [2], while deducing the exact space-time metric representing the exterior of a static, cylindrically symmetric string, employed the Levi-Civita static, cilindrically symmetric metric

$$ds^2 = r^{2m}dT^2 - r^{-2m}[r^{2m^2}(dr^2+dz^2)+a^2r^2d\phi^2] \tag{1}$$

and has obtained

$$ds^2 = dT^2 - dr^2 - dz^2 - a^2r^2d\phi^2 \tag{2}$$

and

$$ds^2 = r^4(dT^2 - dr^2 - dz^2) - a^2r^{-2}d\phi^2 \tag{3}$$

as the metrics corresponding to $m=0$ and $m=2$ from Lorentz invariancy of the string. Equation (2) describes a flat space-time, with or without a conical singularity, depending on the value of a.

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When the interior metric

$$ds^2 = dt^2 - dS^2 - dz^2 - S_*^2 \sin^2(S/S_*)\, d\phi^2 \tag{4}$$

for a uniform density string and (2) are matched together along the junction surface of the string at $S=S_0$, $r=r_0$, they become identical upto coordinate transformations, and result in the exterior field of the string as

$$ds^2 = dt^2 - dr^2 - dz^2 - (1-4\mu)^2 r^2\, d\phi^2 \tag{5}$$

In this note, we use the system of units in which $G=c=\hbar=1$. Metric signature is $(+,-,-,-)$ and $x^i=(t,r,\phi,z)$. It is obvious that for $\mu=1/4$, the string metric reduces to 3-dimensional form, which is not desirable because then there is a change in signature, i.e. signature changes from -2 to -1, which amounts to the change in physics (This we confirm from the study of geodesic motion of a particle in (5) and also the Killing vectors admitted by (5)). We find that the linear density of the string $\mu \neq 1/4$ thus restricting the deficit angle.

2 Trajectory of the Particle

The geodesic motion of a particle in the field (5) is given by time-like geodesics

$$\delta \int \mathcal{L}\, ds = 0 \tag{6}$$

where

$$\mathcal{L} = \dot{t}^2 - [\dot{r}^2 + \dot{z}^2 + \omega^2 r^2 \dot{\phi}^2] \tag{7}$$

and

$$\omega^2 = (1-4\mu)^2 \tag{8}$$

As the coordinates ϕ, z, t are cyclic, the corresponding conjugate momenta will be constant. The resulting Euler-Lagrange equations yield

$$-\ddot{r} = r\omega^2\dot{\phi}^2 \tag{9}$$

$$\omega^2 r^2 \dot{\phi} = H\,(const.) \tag{10}$$

$$\dot{z} = K\,(const.) \tag{11}$$

$$\dot{t} = C\,(const.) \tag{12}$$

where

$$\dot{r} = \frac{dr}{ds} \quad etc.$$

Moreover, the Lagrangian does not involve time t explicitly, therefore the total energy of the system is an integral of motion, given by the Hamiltonian

$$\mathcal{H} = p_t \dot{t} - (p_r \dot{r} + p_\phi \dot{\phi} + p_z \dot{z}) - \mathcal{L} = \mathcal{L} = const. \tag{13}$$

Equations (9) and (10) imply

$$\ddot{r} = -\frac{H^2}{r^3 \omega^2} \tag{14}$$

The transverse component of the acceleration is zero. The force acting on the particle comes out to be

$$F = -m \frac{1}{r^3} \left[\frac{H^2}{\omega^2} (1 + \frac{1}{\omega^2}) \right] \tag{15}$$

and is of attractive nature, and is inversely proportional to r^3.

On integration, (14) gives

$$\dot{r} = \frac{H}{\omega} (\frac{1}{r} - \frac{1}{r_0}) \tag{16}$$

with the initial condition that $\dot{r} = 0$ at $r = r_0$. Furthermore, from (16) after integration, we find that the trajectory of the particle is a parabola in the (r, t) plane. Equations (14), (15) and (16) show that for $\mu = 1/4$, radial acceleration, force and radial velocity all become infinite, which is not acceptable. This restricts the linear density of the string.

3 Killing Vectors

For the metric (5), the Killing equations

$$\xi^k \frac{\partial g_{ij}}{\partial x^k} + g_{kj} \frac{\partial \xi^k}{\partial x^i} + g_{ik} \frac{\partial \xi^k}{\partial x^j} = 0 \tag{17}$$

give

$$\begin{aligned}
&a) && \xi^1,1 && = 0 \\
&b) && r^2\omega^2\xi^2,1 + \xi^1,2 && = 0 \\
&c) && \xi^3,1 + \xi^1,3 && = 0 \\
&d) && \xi^4,1 - \xi^1,4 && = 0 \\
&e) && -\omega^2(\xi^1 r + r^2\xi^2,2) && = 0 \\
&f) && \xi^3,2 + r^2\omega^2\xi^2,3 && = 0 \\
&g) && \xi^4,2 - r^2\omega^2\xi^2,4 && = 0 \\
&h) && \xi^3,3 && = 0 \\
&i) && \xi^4,3 - \xi^3,4 && = 0 \\
&j) && \xi^4,4 && = 0
\end{aligned} \tag{18}$$

Here ξ^i denote the Killing vectors and $\xi^1,1 = \partial\xi^1/\partial x^1$ etc.

These ten equations result into

$$\xi^1 = \omega C_1 \cos\omega\phi + \omega C_2 \sin\omega\phi \tag{19}$$

$$\xi^2 = -C_1 \frac{\sin\omega\phi}{r} + C_2 \frac{\cos\omega\phi}{r} + C_3 \tag{20}$$

$$\xi^3 = C_4 t + C_5 \tag{21}$$

$$\xi^4 = C_4 z + C_6 \tag{22}$$

where C's are constants. Thus, the space time (5) admits the following six Killing vectors

$$\xi^{1i} = (\omega \cos\omega\phi, \frac{-\sin\omega\phi}{r}, 0, 0) \tag{23}$$

$$\xi^{2i} = (\omega \sin\omega\phi, \frac{\cos\omega\phi}{r}, 0, 0) \tag{24}$$

$$\xi^{3i} = (0, 1, 0, 0) \tag{25}$$

$$\xi^{4i} = (0, 0, t, z) \tag{26}$$

$$\xi^{5i} = (0, 0, 1, 0) \tag{27}$$

$$\xi^{6i} = (0, 0, 0, 1) \tag{28}$$

(25), (27) and (28) correspond to infinitesimal translations:

$$\phi \to \phi + \varepsilon$$
$$z \to z + \varepsilon$$
$$t \to t + \varepsilon$$

(26) gives an infinitesimal rotation in (z, t) plane through a complex angle $i\delta a$ about an axis (r, ϕ).

$$As \ \omega \to 0,$$

$$\xi^{1i} \to (0, 0, 0, 0) \tag{29}$$

$$\xi^{2i} \to (0, \frac{1}{r}, 0, 0)$$

resulting in the reduction of the space-time symmetry.

Therefore we conclude that the linear density of a cylindrically symmetric gravitational cosmic string can not be equal to 1/4.

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Electrosorption of Halide Ions in the Pit Initiation of 304L Stainless Steel

C.A.C. Sequeira

Instituto Superior Técnico, Technical University of Lisbon

RESUMO

Estuda-se a corrosão tipo pitting do aço inox 304L por medição de potenciais de rotura, usando a técnica potenciodinâmica. Determina-se o efeito da concentração em cloretiões e da temperatura do electrólito na iniciação das picaduras das amostras metálicas. Para soluções de pH 2 e de temperaturas variando de 20 a 85 °C, obtém-se uma relação linear entre o potencial de rotura e o logaritmo da concentração em Cloreto de Sódio. Também se verifica que o potencial de rotura decresce com o aumento da temperatura.

O cálculo do potencial eléctrico interior dos cloretiões (d.d.p. entre as fases adsorvida e em solução) e da valência de electrosorção, a diferentes temperaturas, mostra que o fenómeno de nucleação está intimamente relacionado com a electroadsorção dos cloretiões, o que permite o uso da isotérmica de Langmuir na interpretação qualitativa dos resultados.

ABSTRACT

The pitting behaviour of 304L stainless steel has been investigated by pitting potential measurements using the potentiodynamic test method. The effect of chloride ion concentration and temperature of the electrolyte on pit initiation in the steel specimens has been determined . In the temperature range of 20 to 85° C and a pH of 2 , the polarization data showed a linear relationship between the pitting potential and the logarithm of sodium chloride concentration. The pitting potential was also found to decrease with increasing temperature.

Calculations of the electrical potential for chloride ions, and of the electrosorption valency, at different temperatures, support the assumption that the nucleation phenomenon is related to the electroadsorption of chloride ions, thus allowing the use of a Langmuir isotherm model for the qualitative interpretation of the results.

1 Introduction

Many workers have studied the effect of aggressive anion concentration on pit initiation (1-7). Most of the investigations relate specifically to chloride ion concentration, but Br^- and I^- concentrations have also been studied. Leckie and Uhlig [1] found a decrease in pitting potential as the NaCl concentration increased. This behaviour was explained by a competitive adsorption process when oxygen and chloride ions compete for sites on surface. They assumed that the amount of anion adsorbed depends on its activity in solution in accord with the Freundlich adsorption isotherm. Manning *et al* [7] found a linear relationship between pitting potential and the logarithm of sodium chloride concentration for 304L stainless steel. Moreover, Lin *et al* [8] provided quantitative explanations of the dependence of pitting potential on halide concentration. They used the concepts of cation diffusion rate and the rate of submergence of cation vacancies into the bulk metal to establish the relationship between pitting potential and activity of halide ion.

Studies of effect of temperature on pitting corrosion have not been done in great detail. Several authors found differently shaped curves of pitting potential versus temperature for various compositions of stainless steels [9-13]. In all cases, increase in temperature decreases the pitting potential toward the active direction, i.e., pitting resistance decreases. Results obtained on the effect of temperature on pitting potential for 18Cr-8Ni stainless steel by Leckie and Uhlig [1], suggest that temperature causes a change in the adsorption kinetics of chloride ions, hydroxyl ions, and water molecules, which results in a change of pitting potential. The results from the studies of Rosenfeld [14] on the kinetics of pit initiation as a function of temperature show that an increase in temperature increases the number of pits. He states that at a higher temperature chemisorption of Cl^- ions on the metal surface are stronger, which causes an increase in the number of pits. Smialowska and Mankowski [15] studied pit growth characteristics change as a function of temperature. They found the kinetics of pit growth was changed by temperature.

A very simple and useful technique for pitting corrosion experiments is the potentiodynamic test, conducted by increasing the potential at a constant rate from the corrosion potential, and monitoring the resulting current [16]. The purpose of the present study is to investigate the pitting corrosion behaviour of 304L stainless steel with respect to its nucleation mechanism, using the potentiodynamic technique. The approach has been to study the effects of environmental variables such as chloride ion concentration and temperature on the potential for pitting initiation. Complementary calculations are included which support the use of a Langmuir isotherm model for the interpretation of the effect of the studied environmental variables.

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2 Experimental Procedure

Details of the experimental procedure were as previously given [17], except as far as the solutions and the polarisation techniques are concerned.

In this investigation, the electrolytes for electrochemical tests were prepared using doubly distilled water with reagent grade sodium chloride and hydrochloric acid.

After the steel electrode was prepared as described [17], the specimen was placed into the corrosion cell. The corrosion potential usually was monitored for fifteen minutes. Then, the electrode potential was adjusted to a potential more active than the corrosion potential by about 100mV to cathodically clean the specimen surface. After this, a potential scan in the anodic direction was made at 0.01V min^{-1} scan rate. The potentiodynamic anodic polarisation curves were recorded for temperatures ranging from 20° C to 85° C, and NaCl concentrations in the test solution ranging from 0.1 to 1.0M . The pitting potential, E_p, was defined as that potential above which repassivation did not occur [18], or, in other words, at the point of rapid current increase the pitting potential was determined experimentally. The pitting potentials reported in this paper are average values obtained from at least five results.

3 Results and Discussion

Figure 1 shows some typical anodic polarisation curves for 304L stainless steel at various chloride ion concentrations, which were used to obtain the pitting potentials.

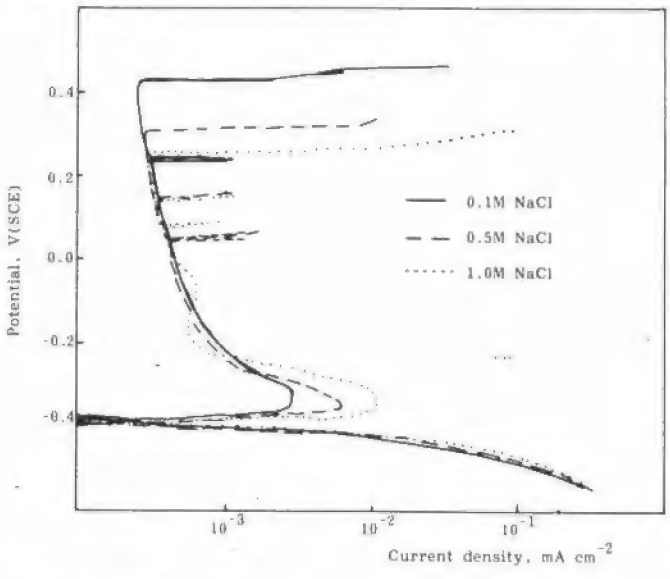

figure 1 Potentiodynamic anodic polarisation curves for 304L stainless steel in NaCl + HCl solutions (pH = 2) at 20°C; sweep rate = 0.01V min^{-1}.

The data in Fig. 2 show a linear relationship between the pitting potential and the logarithm of sodium chloride concentration.

This was measured in the temperature range of 20 to 85°C and a pH of 2. As described in the introduction, several investigators have reported this linear relationship. The data in Fig. 2 also show a linear

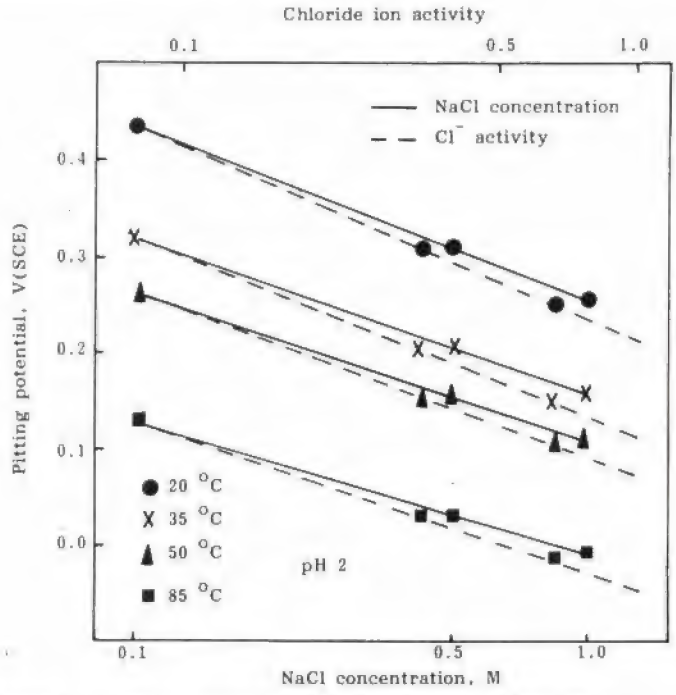

figure 2 Pitting potential of 304L stainless steel as a function of logarithm of sodium chloride concentration and of logarithm of chloride ion activity at various temperature.

relationship between the pitting potential and the logarithm of the chloride ion activity. The proposed equation by Leckie and Uhlig [1],

$$E_p = A - B \log [Cl^-] \quad (1)$$

was applied to the present investigation. The experimentally determined constants A and B are shown in Table 1. The slopes of these equations are very similar. However, the pitting potential decreases with increasing logarithm of sodium chloride concentration.

T, °C	A, V(SCE)	B, V/decade
20	0.215	0.200
35	0.115	0.185
50	0.075	0.173
85	-0.045	0.158

table I Experimentally determined values of the constants in the Leckie & Uhlig Equation, for pH = 2.

The assumption that pitting is related to the adsorption of halide anions is supported by the fact that the introduction of foreign ions can suppress depassivation by halide ions [1, 15, 19-23]. The electric potential at the specimen surface (or externally applied potential) causes the aggressive anions to experience an electrostatic force, which attracts them to the metal surface resulting in electrosorption. A high bulk concentration of the aggressive anions and a sufficiently anodic potential results in adsorption coverage of aggressive anions on the metal surface which in turn initiates pitting on the surface. Therefore, the pitting potential (or externally applied potential) can cause increased aggressive anion concentration on the surface, which leads to a critical surface coverage, θ_c, needed for pit formation and propagation. During pit nucleation electrosorption equilibrium is established between the

adsorbate and the ions in solution. Figure 3 shows a schematic diagram of the double layer with adsorbed chloride ions and water molecules. The electrosorption reaction can be described by the equation [24]

$$MO-Cl^-_{.a}+H_2O+e^-=MO-H_2O_{.a}+Cl^- \quad (2)$$

where MO-Cl_a and MO-H_2O_a represent the specifically adsorbed chloride ions and water molecules at the metal oxide surface, respectively.

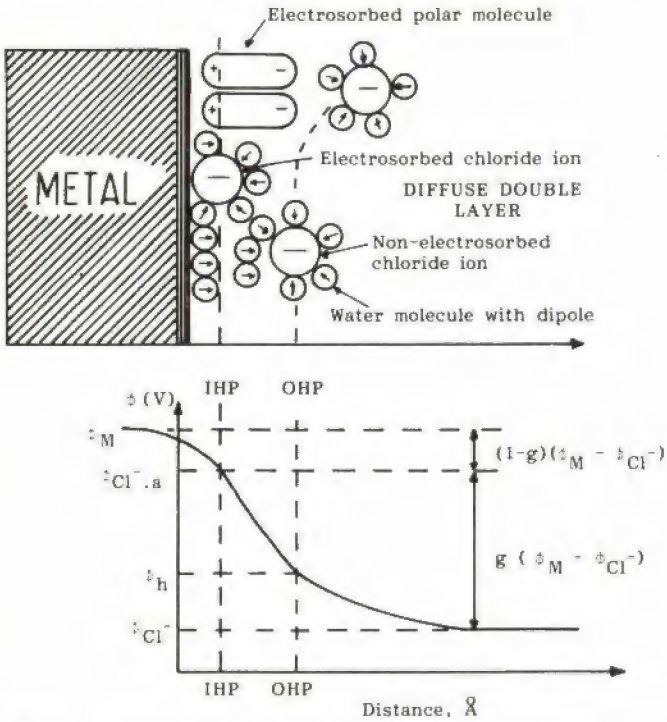

figure 3 Schematic double layer model for electrosorption of chloride ions at metal surface.

Figure 3 also shows that the centers of chloride ions are situated on the IHP (Inner Helmholtz Plane) and the OHP (Outer Helmholtz Plane) and water molecules are adsorbed on the metal surface and are situated around chloride ions. In the case of excess supporting electrolyte the effect of the diffuse electrochemical layer is negligible due to the high ionic strength of the solution, and so the potential difference, ϕ_h-ϕ_{Cl^-}, between the OHP and the electrolyte can be neglected. In other words, the potential ϕ_h becomes the electrical potential for chloride ions in the solution, ϕ_{Cl^-}.

The electrochemical potentials for the chloride ion and water molecule adsorbed and in solution may be expressed by the following equations:

$$\bar{\mu}_{Cl^-_{.a}}=\mu^o_{Cl^-_{.a}}+RT\ln\theta+zF\phi_{Cl^-_{.a}} \quad (3)$$

$$\bar{\mu}_{Cl^-}=\mu^o_{Cl^-}+RT\ln[Cl^-]+zF\phi_{Cl^-} \quad (4)$$

$$\bar{\mu}_{H_2O_{.a}}=\mu^o_{H_2O_{.a}}+RT\ln(1-\theta)+zF\phi_{H_2O_{.a}} \quad (5)$$

$$\bar{\mu}_{H_2O}=\mu^o_{H_2O}+RT\ln[H_2O]+zF\phi_{H_2O} \quad (6)$$

where θ is fractional surface coverage, μ^o and [ ] are the standard chemical potential and activity of the species, respectively.

For the electrosorption equilibrium of reaction [2] it follows

$$\bar{\mu}_{Cl^-_{.a}}-\bar{\mu}_{H_2O_{.a}}=\bar{\mu}_{Cl^-}-\bar{\mu}_{H_2O} \quad (7)$$

The difference in the apparent electrochemical free energies of adsorption of chloride ions and water molecules is given by

$$\Delta G=(\bar{\mu}_{H_2O_{.a}}-\bar{\mu}_{H_2O})-(\bar{\mu}_{Cl^-_{.a}}-\bar{\mu}_{Cl^-}) \quad (8)$$

Neglecting the field effect terms for the water molecules due to their neutral charge, with [7] we obtain from [8] the potential difference between the adsorbed and non-adsorbed chloride ions,

$$\phi_{Cl^-_a}-\phi_{Cl^-}=\frac{1}{zF}\left\{\left[(\mu^o_{H_2O_{.a}}-\mu^o_{H_2O})-(\mu^o_{Cl^-_{.a}}-\mu^o_{Cl^-})\right]-RT\ln\left(\frac{\theta}{1-\theta}\right)+RT\ln[Cl^-]\right\} \quad (9)$$

We may write

$$\phi_{Cl^-_a}-\phi_{Cl^-}=g(\phi_M-\phi_{Cl^-})=g\,\Delta\phi \quad (10)$$

where g is a geometric factor describing the penetration of the adsorbate into the electric field of the double layer, and $\Delta\phi$ is the potential difference between the metal surface and the outer layer. Since the chloride adsorption is an electrosorption process of the type (2), a more meaningful physical concept can be given by the introduction of the electrosorption valency, γ (25). In the case of excess supporting electrolyte, γ is defined by the equation

$$\gamma=-\frac{1}{F}\left(\frac{\partial q_m}{\partial\Gamma_a}\right)_E=\frac{1}{F}\left(\frac{\partial\mu}{\partial E}\right)_{\Gamma_a} \quad (11)$$

where q_m is the electrode charge, Γ_a the surface concentration of the specifically adsorbed substance, μ is the chemical potential in the solution, and E is the electrode potential ($E=\Delta\phi$).

The term "electrosorption valency" was chosen because of the analogy between the value, γ, and the electrode reaction valency, z, which enters into Faraday's law as well as the Nernst equation. The analogy between an electrosorption reaction (2) and any electrode reaction can be demonstrated with the case of deposition of metal ions from the solution on a liquid alloy. For this reaction, the Nernst equation

$$E=E^o+\frac{RT}{zF}\ln\frac{[Ions\ in\ electrolyte]}{[Metal\ in\ alloy]} \quad (12)$$

yields the following derivative in analogy to equation (11)

$$\left(\frac{\partial \mu}{\partial E}\right)_{[M\ in\ alloy]} = zF \qquad (13)$$

where z is the electrode reaction valency.

The deposition of n moles of metal ions requires a charge, q_m. Taking into account changes of the potential, ΔE, it follows

$$q_m = -zFn + C_D \Delta E \qquad (14)$$

where C_D is the capacitance of the double layer. From (14) we obtain Faraday's law by differentiation at constant potential:

$$\left(\frac{\partial q_m}{\partial n}\right)_E = -zF \qquad (15)$$

Equation (15) again is analogous to equation (11). However the value of z in Faraday's law is always an integer, whereas γ can be a fractional number. The electrosorption valency controls the potential dependence and the charge flow in the same way as z does in the case of an electrode reaction, e.g. the metal deposition. It can be stated that the electrodeposition is equivalent to an electrosorption with complete charge flow, $\gamma = z$.

The potential drop across the double layer, ΔE, can be expressed by the difference of the potential of the metal oxide surface, ϕ_M, and the drop in the double layer of the standard hydrogen electrode, ϕ_{SHE}. When no charge transfer takes place during adsorption, $g = \gamma$. Thus, equation (10) can be rewritten as,

$$\phi_{Cl^-_a} - \phi_{Cl^-} = g\,\Delta\phi = \gamma\,\Delta\phi = \gamma(\phi_M - \Delta\phi_{SHE}) \qquad (16)$$

This equation indicates that the chloride ions move toward the metal surface in accordance with the term $\gamma\Delta\phi$ difference and do the electrostatic work $z\,F\gamma\Delta\phi$ per mole during the adsorption process.

Inserting the critical values $\phi_M = E_p$ (pitting potential), $\theta = \theta_c$ and $z = -1$ into equations (9) and (16), and rearranging them, it follows

$$E_p = A - B\,log\,[Cl^-] \qquad (17)$$

where

$$A = \frac{1}{\gamma F} + \left[(\mu^o_{Cl^-_a} - \mu^o_{Cl^-}) - (\mu^o_{H_2O} - \mu_{H_2O})\right] + \frac{2.3\,RT}{\gamma F}\,log\left(\frac{\theta_c}{1-\theta_c}\right) + \Delta\phi_{SHE}$$

and

$$B = \frac{2.3\,RT}{\gamma F}$$

According to equation (17) the pitting potential decreases linearly with the logarithm of the chloride ion activity. The value may be determined from the slope of the experimental plot of equation (17),

$$\frac{\partial E_p}{\partial\,log[Cl^-]} = \frac{-2.303\,RT}{\gamma F}$$

In this investigation, the electrosorption valencies were determined to be:

$$\gamma_{20^oC} = 0.29 \qquad \gamma_{50^oC} = 0.37$$

$$\gamma_{35^oC} = 0.33 \qquad \gamma_{85^oC} = 0.45$$

The electrosorption valency was found to increase with increasing temperature. The significance of this increase may be elucidated by a consideration of the change in pitting potential with temperature and the potential energy diagram (Fig. 3). The pitting potential was found to decrease with increasing temperature, i.e. 260 mV at 20°C, 165 mV at 35°C, 110 mV at 50°C and -10 mV at 85°C in 1.0M NaCl electrolyte.

The free energy change of the chloride ion with temperature is

$$\Delta G = n\,\mu^o_{Cl^-} + n\,RT\,ln[Cl^-]$$

where n is the number of moles of NaCl. For 1.0M NaCl electrolytes, the chloride ion activities are 0.656, 0.660, 0.658 and 0.636 at 20°C, 35°C, 50°C and 85°C, respectively, these values deriving from $log\,f = 11.4326 - 535.45/T - 3.9679 log T$, where T is absolute temperature, and f is activity coefficient. Therefore,

$$\Delta G_{20} = \mu^0_{Cl^-} - 245.45 \qquad \Delta G_{50} = \mu^0_{Cl^-} - 268.63$$

$$\Delta G_{35} = \mu^0_{Cl^-} - 254.29 \qquad \Delta G_{85} = \mu^0_{Cl^-} - 321.92$$

The changes in the ϕ_{Cl^-} with temperature were calculated to be :

$$[\Delta\phi_{Cl^-}]^{35}_{20} = -(\Delta G_{35} - \Delta G_{20})/nF = 0.383\ mV$$

$$\left[\Delta\phi_{Cl^-}\right]^{50}_{35} = 0.622\ mV$$

$$\left[\Delta\phi_{Cl^-}\right]^{85}_{50} = 2.31\ mV$$

Comparing the change in pitting potential at a specific temperature to that in ϕ_{Cl^-}, the change in ϕ_{Cl} was found to be negligible. Thus, the change in pitting potential, E_p, or the metal potential, ϕ_M, with temperature is directly related to the magnitude of change in potential of the metal surface with temperature as can be seen from Fig. 3.

The electrosorption valency, γ, is a measure of the fraction of the potential difference, ϕ_M-ϕ_{Cl^-}. Although γ increases with temperature, the energy barrier between the metal and solution is decreasing rapidly. Consequently pitting corrosion occurs more readily at higher temperatures than at lower temperature due to the lowering of ϕ_M with temperature. The effect of temperature on pitting potential in various NaCl concentrations with pH = 2 is also illustrated in Fig. 2. As temperature increases, the pitting potential decreases. Moreover, 304L stainless steel pits at lower potentials in strong chloride ion concentration than in weaker chloride ion concentration. Because of a stronger

dependence of pitting potential on temperature than on chloride ion concentration, a critical pitting temperature can be defined, as proposed by Brigham [13], at which the stainless steel would not pit regardless of potential and exposure time. An increase in the temperature affects the pitting process in a number of ways other than that associated with composition changes in a protective film. Probably by reducing the density and viscosity of the original aqueous solutions, the mass transfer coefficient will be increased and the diffusion rate of dissolved species controlling the corrosion rate (pitting) will be increased, i.e. oxygen, chloride ions and metal ions.

The effect of pH and other environmental variables on pitting corrosion for 304L stainless steel is being studied and will be published shortly. Preliminary results on the pH effect could also be interpreted qualitatively using a Langmuir isotherm model. This further supports our treatment of pit nucleation in terms of an electrosorption model.

4 Conclusions

1. A linear relationship exists between the pitting potential and the logarithm of chloride ion activity and is given by

$$E_p = A - B \log [Cl^-]$$

2. The electrosorption valency, γ, increases with increasing temperature, i.e. 0.29 at 20°C, 0.33 at 35°C, 0.37 at 50°C, and 0.45 at 85°C.
3. When the change in E_p at specific temperature was compared to that in ϕ_{Cl^-}, the change in ϕ_{Cl^-} was found to be negligible.
4. The pitting potential decreases with increasing temperature, i.e. 310 mV (vs SCE) at 20°C, 205 mV at 35°C, 155 mV at 50°C, and 40 mV at 85°C in 0.5M NaCl electrolyte.
5. Pit nucleation can be described by electrosorption processes of the type

$$MO - Cl^-_{.a} + H_2O + e^- = MO - H_2O_{.a} + Cl^-$$

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AQUI PODERIA ESTAR UM ANÚNCIO SEU

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Cybernetics and Possibility Theory

The Limitations of the Classical Paradigm

Pedro Roberto Medina Martins

Departamento de Mecânica , Secção de Sistemas, Instituto Superior Técnico, UTL

RESUMO

Indicam-se possíveis vias para resolver os problemas levantados pelas insuficiências do paradigma clássico no tratamento de sistemas respeitantes às ciências humanas.

ABSTRACT

An attempt is made to tackle the problems arising out of the inadequacy of the classical paradigm in dealing with humanistic systems.

1 Introduction

Those who, for quite a long time, have undergone the influence of a particular culture, society or community and have often tried to discover what their origins were as well as along what path they have developed are, most of the times, tempted to take a glance in the other direction and to ask what further fate lies before them and what further transformations they may be subjected to. However (and contrarily to what usually occurs in the so-called 'hard sciences' in which some kind of anticipation, prediction, prospection or forecasting is, in principle, always possible) those who are engaged in such a task soon realize that the value of their inquiry is from the outset, dwindled by several factors. Amongst them the follwing four are especially poignant.

Firstly, and above all, no one can (presently) survey human activity in its full compass. Even belonging to some interdisciplinary team, people are obliged to restrict themselves to a single or few fields of it (which, in turn, will raise problems of interpersonal communication, data interpretation and so forth) leaving for some (hypothetical) team coordinator the onus for interpreting the partial conclusions which the team's members have obtained. This fact not only enhances the argument but also begets a second difficulty.

As a matter of fact, the less we know about the past and the present the most insecure (and uncertain) must prove to be our judgements about the future; although the same objection can be stated as regards the possible temporal evolution of systems which 'hard' sciences study, those which 'soft' disciplines aim for beset an additional and hardly overcoming trouble.

Precisely, that in such a kind of judgements the subjective expectations of the individual play a role whose influence cannot be altogether disavowed. As a further complication the assessment to such expectations is extremely difficult. They depend, to a great extent, on pure personal factors in his/her own experience, on the greater or less optimism and pessimism to life as they have been dictated by his/her temperament, by the success or failure of his/her experiences/experiments, etc., of which one usually remains unware.

A third factor, not directly related to the individual's past, makes itself felt when people are experiencing their present. Generally speaking, this experience has a naive character as though people have first to put themselves at a distance from it (situation in which the present must have become the past) before it can yield points of vantage from which the future can be judged - a fact which never occurs in the observation of 'hard' systems.

Finally, the fourth factor deals with the proper observed culture, society or community now not taken as wholes (as so far they have been altogether regarded) but, rather, as collections of highly distinctive, idiosyncratic and personalized *qua sentient* beings. Human beings are, in reality, odd creatures who not only want to be loved and who, at the most, can defend themselves if they are attacked but also creatures among whose instinctual endowment is to be reckoned a powerful share of aggressiveness and violence. Anyone who looks backwards to the whole history of mankind can see that the civilization to which we presently pertain has been erected at the cost of the destruction of countless lives. Against the sudden eruption of these forces which, ultimately, threaten either the physical integrity of the individual with anihilation or the communal life with desintegration, civilization (and thenceforth, because meaningless in this context, I shall contemn the distinction between civilization and culture) has, throughout time, improved several types of regulations, institutions and commands. I shall not enumerate them all but, together with the judicial right for using violence against those considered as criminals, methods intending to incite people into identification, interests of work in common, religious or ideals commandments (remember the world-demand "Thou shall love thy neighbour as thyself"),etc., are for instance, some of the most commonly used. It seems, however, that despite everything already gained in the

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substitution of leaderships based upon the brute force of some individual by the collective power of communities founded on right and law (more or less) freely accepted, despite the increasing tendency for the realization of that 'global society' always dreamed about men and women of all epochs, these endeavours (and hopes) of civilized societies have not so far achieved very much. Men, women and children are, nowadays, yet killed all around in the name of some ideology, religious fanatism, privation of essential surviving goods, simply because they belong to 'another' football team or even in the name of love and peace.

I shall not discuss here the possible philogenetic, ethological, roots of such behaviours. For the moment (and although people are generally ready to disavow them) I shall only stress that, even under the more cautious, desguised and refined forms, these manifestations of human aggressiveness willy nilly exist in each of us and that (as a rule) they work as though they were waiting for some provocation to be externalized. If the circumstances are favourable, say, if the mental counter-forces which ordinarily inhibit them are out of action, then they manifest themselves spontaneously with the results we all presently know.

The existence of these forces (the strenght of which we are usually unaware) introduces an additional element of uncertainty in the possible evolution which the observed culture, society, or community may undergo. Phrased metaphorically, I would say that their role is tantamount to that played by the 'hidden parameters' of some *locally* unstable (albeit *globally* stable) dynamical 'hard' system, so that a minor variation in one of their values at any (historical) instant may lead its state-trajectory to a catastrophe (bifurcation and so forth) in Thom's sense.

If this (long-time) unpredictableness of human societies is added to the three remainder factors so briefly delineated in the foregoing considerations, we can see how the building up of a 'theory' of human actions (individually or socially regarded) is far from being achieved. In spite of this - and this is one of the major contentions of this work - I do believe that such an endeavoured objective is reachable. This, if the progress, enhancement and transformation which science (in general) has undergone in the last fourty years is *appropriately* taken into account. The emphasis laid on '*appropriately*' is here crucial for it besets two, albeit dependent, fundamental questions (closely associated with the matters to which my attention has so far been directed) which, briefly, may be stated as follows:

a) Why the broad mainstream of present-day science (based upon the widely held belief that precise, quantitative, reasoning is *the* means to solve complex problems) has, in some sense, been unable to provide answers to the ill-defined problems that the analysis of humanistic systems underlies?

b) Can such answers be (partially, at least) obtained from new approaches or new perspectives which, though pertaining to the present-day scientific body of knowledge, contravene that belief in some way and, as such, are less known or less adopted?

The answer to these questions is far from being easy. But, to summarize my opinion about them, I would say as regards question a) the it entails not only the knowledge of the ultimate standpoints on which that broad scientific mainstream is based but also *the awareness of its proper limits and insufficiencies* (which is much more difficult to realize and accept, particularly from the side of those who, for quite a long time, have been engaged in the mental framework it underlies). Once those standpoints, limits and insufficiencies are clearly established then I shall demonstrate how some 'less known' subjects as second order cybernetics, fuzzy sets and possibility theory, etc., as well as their applications may provide (possible) solutions to question b).

In order to organize these matters the work will be subdivided into several, albeit mutually related, monographs. Monograph 1, the present one, will be devoted to the analysis of the main reasons on which the present-day gap between 'hard' and 'soft' sciences lies. In Monograph 2, I shall present the features of relativistic paradigms which, to a great extent, provide answers to the problems which the paradigm followed by the mainstream of present-day sciences raises when applied to the modelling and simulation of mental phenomena. A specific (second order) cybernetic application of these relativistic paradigms to social and conversational schemes will be delineated in Monograph 3. Finally, in Monograph 4 some advanced topics in Conversation Theory will be approached by means of fuzzy sets and possibility theory.

2 The Gap Between 'Hard' and 'Soft' Sciences and the Classical Paradigm

The debate between the followers of the "hard" and "soft" sciences (a sort of epistemological 'no mans land') together with everything it subsumes not only in terms of controversies and divergences but also in promising research lines still overt to further exploitation has been recently renewed thanks to the remarkable development which the so-called 'knowledge engineering' has undergone in the last two decades (a development which, in turn, is a by-product of all the technological changes that have been produced in the computational area). Its sources are, however, far more ancient being firmly rooted in the whole history of the Western civilization. Implicit allusions to it may already be found either in the ancient Greek thought (remember, among other examples, the discussions about what is 'natural' or 'product of men', or the controversy 'rational' versus 'irrational'). More explicit hints are due, however, to the movement which, beginning with Nicolas de Cusa, Giordano Bruno, Copernicus, Kepler, Galileo, Descartes, Newton and others, has progressively led to the substitution of the Greek Image of Nature by the Renaissantist one - an image where Nature was no longer regarded as an organism (with a 'mind', a 'body' and a 'soul ' somehow identified with the God of the Christian religion) but, rather, as a machine (or, more corrrectly, as a causal sequential machine) ruled by an external, transcendent, God.

For the moment I shall not go into minute discussions about the ultimate motives or consequences of this revolutionary change upon the image which Man has, thenceforth, progressively constructed either of his "natural" or "physical" world or of himself, both as individual and as member of human communities. However, for my present purposes, two of such consequences deserve special attention. On the one hand, that the effective success of the mechanist paradigm in the explanation of natural phenomena (until the late decades of the XIX th century, at least) has later begotten not only what A. Rey called "the consciousness of the definitive" in physics but also the birth of a way of thinking (soon converted in a true scientific ideology) in which the methods and assumptions of this discipline were supposed to be integrally extended to the full understanding of the biological, psychological and social entity which the human being is. On the other, and this is a crucial point, that this ideology (thenceforth named 'classical paradigm' for the sake of conciseness) is still framing the status of a great deal of present-day disciplines, this in spite of its proved failure in the realm of the contemporary physics (the status of the classical paradigm is now simply confined to the observation and description of coarse, grained phenomena). We all know, indeed, how a language based upon such words and expressions as 'systems', 'objects', 'environments', 'stimuli', 'causative inputs', 'responses', 'outputs', 'transfer functions', 'external, impartial (and sometimes also omniscient) observers', 'estimation' and the like is pervading the present-day literature, even the non-expertise one.

Although in engineering and physical/quasi-physical sciences the fruitfulness of the logical-mathematical techniques which these expressions subsume cannot be denied, some other areas exist, however, where their using has been not only restrictive but also regretably baneful. One of such areas embraces, precisely, the disciplines where mentation in general (attention, thinking, willing or striving and so forth) as well as its externalized expressions (utterances, gestual, body-languages, etc.) are the main focus of attention.

The reasons for this situation are manifold. On the one hand, they deal with all the ubiquitous problems which the researchers have to face when treating of the real complexity of the human mind's functioning. On the other, however, they also proceed either from the proper characteristics of the formal approach on which such techniques have generally been based (particularly in the last fifty years) or of its *interpretation* and *use*, both of which are aspects whose importance has been often over-looked.

The approach referred to beforehand is presently connoted with the so-called "General Systems Theory" (Klir, Bunge, Mesarovic, Kalman, Gougen and so forth). General System Theory is the contemporary inheritor of those positivist and neopositivist schools of thought which, with Schlik, Carnap, Wittgenstein, Russel, Whitehead, Lukasiewcz and others, overemphasized in the beginnings of this century the role of mathematical logics upon pratically all the domains of the human knowledge. As (supposedely) unified language for the description of a great deal of disciplines, the present-day role of the systemic approach (and of its cognate paradigm) become, to this extent, tantamount to that played by the late versions of the pure mechanistic framework.

As previously stated, I do not dispute the usefulness of General Systems Theory in engineering or physical and quasi-physical sciences. But let me discuss more circumstantially the reasons as to why its using as a general framework either for mind sciences (here including its externalized expressions) or, even, for the taxonomy of sciences has often given rise to serious, albeit false, problems.

2.1

In the realm of mind sciences, this framework has, in fact, led to a rather artificial separation between what is (supposed to be) 'cognitive' (or, commonly, *rational*) and 'affective' (or, commonly, *irrational*) as though the mind's functioning could be arbitrarily straddled and everything related to affects, emotions, feelings and their underlying processes could be supressed. Yet, according to the neopositivist thought, almost exclusive emphasis has been laid on the first in detriment of the second (an idea springing from the argument that such a cognitive functioning is 'objective', thus, 'concrete' and 'reliable', whereas the affective one is 'subjective', thus, 'fallible' and consigned to non-standard schemes such as introspection, retrospection and so forth). Because of this (and amongst other consequences) the polimorphous/dispositional concept of thinking (from Ryle's perspective) has also been almost exclusively identified with 'reasoning' (in the sense of the Cartesian "cogitatio"). Hence, an additional (over)emphasis upon deductive reasoning, problem-solving and truth-searching methods, a research line which the defenders of the so-called Artificial Intelligence (to some extent, the most direct inheritors of the mechanistic ideology) are fully exploring.

For the moment I shall not go into details about this particular way of looking upon computers' using as well as to its (endeavoured) extension to the simulation and, even, (intended) description of the whole mentation. But let me stress out only a couple of critical contentions one of which recalls Ryle's conception of thinking (attention, striving, and so forth will be deferred to later comments), the another interactions in general (and Man/machine in particular) when regarded from the classical perspective.

2.2

In connection with the first of such criticisms you must bear in mind that, together with 'reasoning', thinking does also deal with what is named 'autistic thinking' (a general designation entailing underlying whishes, needs, wants, etc., of which fantasies, day-dreams, the idle-flitting from one half-formed notion to another are common psychical expressions), 'imagination', 'remembering', 'styles of behaviour' and 'beliefs'. However (and this is the crux of the argument), with exception (perhaps) of the epistemic beliefs, none of

the remainder modalities of thinking can be reduced either to 'pure deduction from previously known (or implemented) premises' or even to logic (in the classical sense, at least). On the contrary, and the following listing is by no means exhaustive, what fantasies, day-dreams and the like overemphasize is (amongst other things) the existence of a stream of autonomous responses to internal changes (unlikely to be reduced to classical logic and pure deduction) whereas remembering, styles of behaviour and deontic beliefs carry the stamp of the living experiences of which our personal history is made. Thus, embracing also time (regarded here as a diachronic dimension), maturation, development, growing, evolution, learning and so forth as well as social interactions - all of which are far beyond the scanty identification of thinking with (logical) deduction. Even 'reasoning' in the sense of 'reflecting', 'pondering' or 'cogitating' (what the old writers used to call 'intelect') is highly ambiguous in meaning. Usually, the term is connoted with the activity of going over a neat argument which has already been worked out and accepted on a previous occasion. No one denies this. But, as Ryle and Thomson show, this is only part of the story. Consider, for example, the case of a thinker indulged in some sort of reflective thinking and to whom a (retrospective) description of his activities is asked. He may tell us that he has been working through a series of calculations (using, for instance, a calculating machine or a computer). He may say that he has some kind of intuition as regards a particular conclusion he has been striving for and then that he has constructed a chain of inferences to deduce this conclusion from well-established premises; or, conversely, that he had been led to the 'formation of hypotheses' in terms of which a certain range of events he has been working out could be explained (which, as McCulloch demonstrated, is neither a deductive nor an inductive activity but a mix of abduction/distinguishing/inventing). He may also tell us that he has been asking himself specific question and giving answers in the light of facts and theories known and remembered. Or, that he has been working as though he were arguing a case - taking first one and then the opposite side in the issue; or, that he has been posing cross-questions to himself (playing the roles of 'witness' and 'counsel') in order to solve some ambiguity or conflict. Or, yet, that he has been consulting books, papers and mathematical tables in order to complete some inference (all of which are activities motivated towards specific goals and, to a great extent, also determined by previous learning and experience), etc. All of these activities may be regarded as part and parcel of 'reasoning' but none of them, taken separately, draws a cogent picture of what a thinker does as he/she is reasoning. Hence, the reason why Ryle refers to it as a 'dispositional' concept, something involving a complex process of coordination and integration of specific activities which (as Thomson asserts) "may be likened to the account of a battle where moves and counter-moves, tactics to be adopted and their modification in the light of obstacles, contingencies and requirements imposed upon by the situation, methods of attack to be employed and results of his/her strivings" are, permanently, being deemed.

I won't go into more details about this criticism. However, for the sake of clearness, two conclusions may be outlined from it. That 'thinking' (even in the sense of 'reasoning' or 'cogitating') is, essentially, an activity or performance consisting in the exercise of skills and capabilities in a rather special way far beyond pure deduction; and, that the kind of performance involved (i.e, the particular skills and the manner or style according to which they are executed) depends not only on the context into which each individual is inserted but also on his/her highly personalized idiosyncratic features. The importance of these points will, later, be explicated.

2.3

The second of the critical contentions that I had previously stressed out was referred to the perspective from which interactions in general (of which those concerning Man/machines are rather particular cases) are encompassed by the classical point of view.

An aspect which will be crucial for the comprehension of this criticism is my previous assertion about the physical sources of that perspective. Let me explain it more circumstantially. Generally speaking, physics is a discipline which, since its beginnings, has aimed the 'description' of natural phenomena. Until the XVII th century (with exception of Astronomy) ordinary language was, pratically, the unique systematic tool used for achieving such a purport. However, with the rising up and further development of the Renaissantist Image (particularly with Kepler, Galileo, Newton, Descartes, Leinitz and followers), the ultimate grounds on which those descriptions had, for centuries, been based underwent profound transformations. From there on, not only the very nature, the gist of such a 'language' becomes (almost) exclusively mathematical but its descriptive power also acquired a new (ever dreamed) feature: the ability for making predictions, the confirmation of which was/is (presently) interpreted as demonstrating either the adequacy between the model constructed and the (natural) 'reality' (say, in other words, of the 'truthfulness' of the correspondences model/reality) or the cogency of the model itself, of its underlying premises, deductions, etc. In addition to these changes, the Renaissantist Image also introduced (and later improved to its extreme consequences) the so-called 'experimental method', the ultimate means which, since then, physicists have at their disposal to confirm (or to deny) such a desired 'truthfulness'.

As I mentioned before the real success of these standpoints in the explanation of great many deal of natural phenomena was in the origin of the (supposedely) whole-embracing scientific framework whose extension to the psychological description of human beings I have been mooting. To this purpose let us examine more carefully some of the (pre)supositions that the framework encompasses. From them, those related to its implicit measurable aspects (implying a distinction between someone who measures and